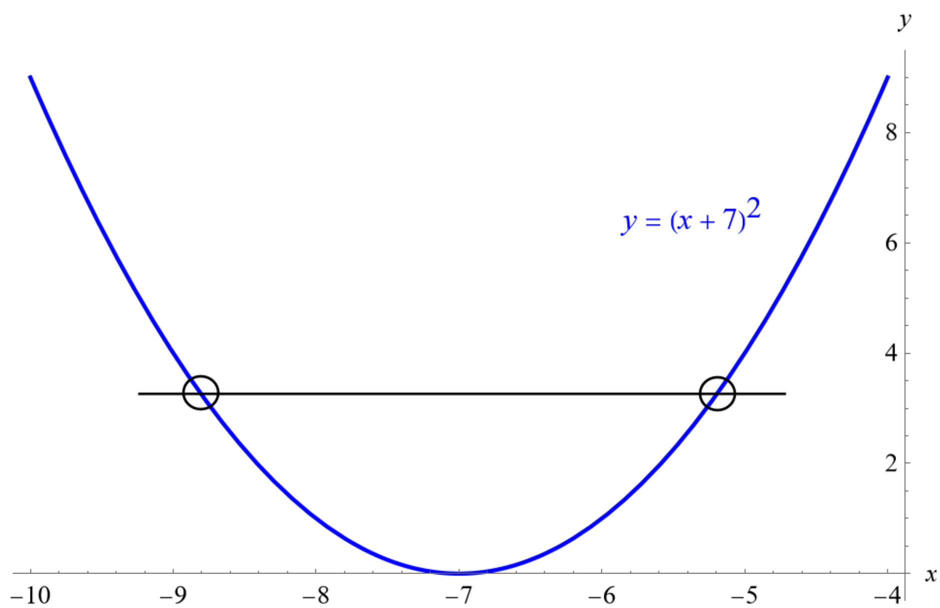
Exercise 13

For the following exercises, find a domain on which each function f is one-to-one and non-decreasing. Write the domain in interval notation. Then find the inverse of f restricted to that domain.

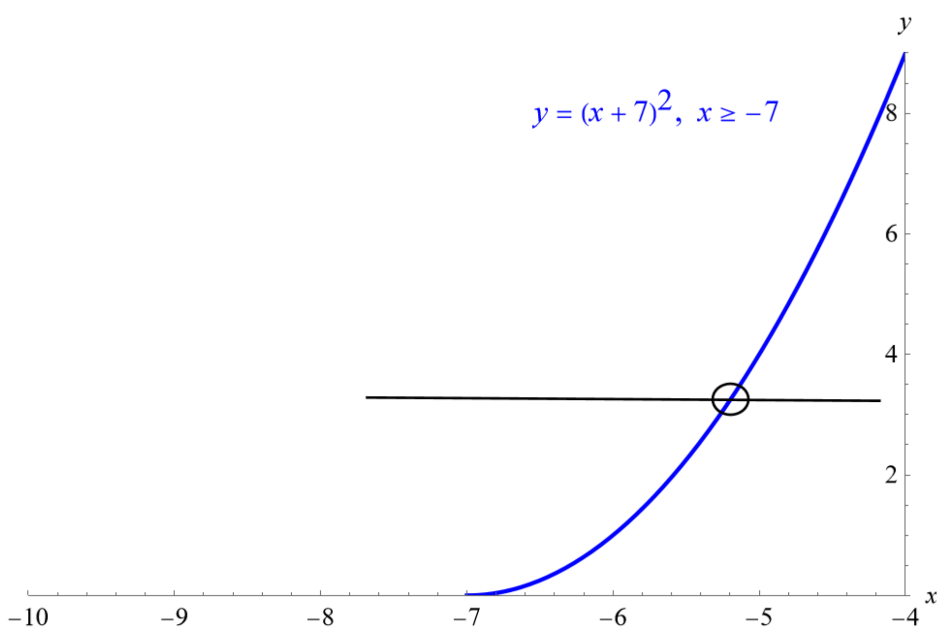
$$f(x) = (x+7)^2$$

Solution

This function is not one-to-one because it fails the horizontal line test.



But it can be made one-to-one by taking the restriction of $f(x)$ to $x \geq -7$.



The domain on which $f(x) = (x+7)^2$ is one-to-one and non-decreasing is $[-7, \infty)$. To find the inverse, switch x and y.

$$x = (y+7)^2$$

Solve for y. Take the square root of both sides.

$$\sqrt{x} = \sqrt{(y+7)^2}$$

Since there's an even power under an even root and the result is odd, an absolute value sign is needed.

$$\sqrt{x} = |y+7|$$

Remove the absolute value sign by placing $\pm$ on the left side.

$$\pm\sqrt{x} = y+7$$

Subtract 7 from both sides.

$$y = \pm\sqrt{x} - 7$$

In order to decide whether to choose the plus or minus sign, notice that y originally came from x, which has the domain $[-7, \infty)$. Choosing the minus sign would allow values of y less than -7. Therefore, the inverse function is

$$f^{-1}(x) = \sqrt{x} - 7.$$